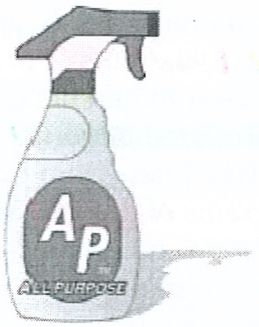
Example 2: Solve an Applied Maximization Problem

A company manufactures two types of cleansers. One is an all-purpose cleanser (*AP*) and the other is an industrial strength cleanser (*IS*). Each cleanser is a mixture of three chemicals, as shown below.



All-purpose, AP
Each kiloliter requires:
12 liters of surfactants
9 liters of enzymes
30 liters of solvents

Industrial strength, IS
Each kiloliter requires:
24 liters of surfactants
5 liters of enyzmes
30 liters of solvents

The profit per kiloliter from the *AP* cleanser is \$100 and the profit per kiloliter from the *IS* cleanser is \$85. The inventory of the company shows 480 liters of surfactants, 180 liters of enzymes, and 720 liters of solvents available. Assuming the company can sell all the cleanser it produces, find the number of kiloliters of each cleanser the company should produce to maximize profit. What is the maximum profit?

Step 1: Define the variables.

x = # of KL of AP

y = # of KL of IS

Objective function: $P = 100x + 85y$

Step 2: Write a system of inequalities.

$$\begin{cases} 12x + 24y \le 480 \longrightarrow x + 2y \le 40 \longrightarrow y \le -\frac{1}{2}x + 20 \\ 9x + 5y \le 180 \longrightarrow y \le -\frac{9}{5}x + 36 \\ 30x + 30y \le 720 \longrightarrow x + y \le 24 \longrightarrow y \le -x + 24 \\ x \ge 0,\ y \ge 0 \end{cases}$$

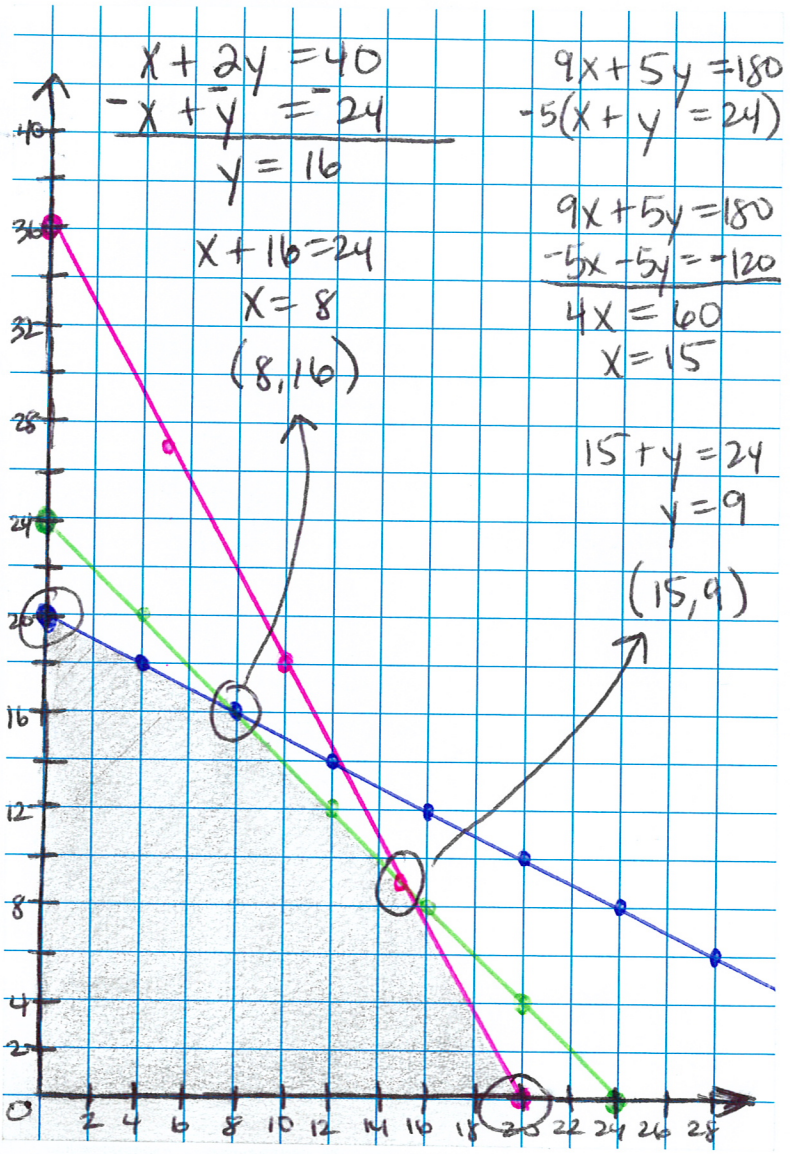
$(0, 20)$: $P = 100(0) + 85(20) = 1700$

$(8, 16)$: $P = 100(8) + 85(16) = 2160$

$(15, 9)$: $P = 100(15) + 85(9) = 2265$ ★

$(20, 0)$: $P = 100(20) + 85(0) = 2000$

The company should produce 15 KL of AP cleanser and 9 kL of IS cleanser to get a maximum profit of \$2265.


x + 2y = 40
-x + -y = -24
y = 16
x + 16 = 24
x = 8
(8, 16)
9x + 5y = 180
-5(x + y = 24)
9x + 5y = 180
-5x - 5y = -120
4x = 60
x = 15
15 + y = 24
y = 9
(15, 9)
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
4
8
12
16
20
24
28
32
36
40